Correcting Arbitrary Hybrid Defocus and Astigmatism for Near-Eye Displays Using Two-dimensionally Displaced Alvarez Lenses

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Abstract

This paper proposes an Alvarez lens for vision-correcting near-eye displays. A novel two-dimensional displacement approach with independently controlled direction and distance is proposed to correct arbitrary hybrid defocus and astigmatism. The proposed Alvarez lenses are experimentally verified on an Apple Vision Pro using various combinations of defocus and astigmatism.

Author Keywords

Near-eye display; Prescription correction; Alvarez lens.

1. Introduction

Near-eye displays (NEDs) are essential for VR/AR; however, at least 2.2 billion people worldwide are suffering from refractive error, significantly hindering the acceptance of NEDs because wearing a NED with spectacles is uncomfortable or even cannot be accommodated by the NED's exit pupil. The refractive errors primarily include defocus (myopia/hyperopia) and astigmatism. Since each user has a different type and extent of refractive error, it is necessary to correct the ametropia for the proper use of NEDs.

Unfortunately, NEDs are usually composed of fixed-power lenses. One solution to vision correction is prescription-customized optics [1, 2]. For instance, the Apple Vision Pro requires users to order Zeiss Optical Inserts based on their ophthalmic prescriptions. However, different prescriptions require different optics, significantly increasing the cost and reducing flexibility.

Another approach is varifocal elements for dynamic correction, such as liquid crystal (LC), liquid, electro-wetting, and Alvarez lenses [3-6]. However, these traditional varifocal elements can only correct myopia and hyperopia but cannot address astigmatism. Further, Bhowmick et al. proposed a tunable lens system consisting of three stacked cylindrical LC lenses with adjustable curvatures [7]. Qiu et al. proposed that the light field display can achieve computational vision correction by manipulating vectorial rays [8]. However, using the LC lens or similar varifocal elements to correct astigmatism is challenging, let alone arbitrary hybrid astigmatism and defocus. The light field display-based solution cannot work for ordinary non-3D NEDs.

Regarding the above shortcomings, we propose a novel two-dimensionally (2D) displaced Alvarez lens pair, which can correct arbitrary hybrid defocus and astigmatism. By contrast, conventional Alvarez lenses are only one-dimensionally (1D) displaced for a tunable focal length. The compensated wavefront corresponding to the 2D displacement is an arbitrary hybrid of defocus and astigmatism. Additionally, the axis of astigmatism is configured by rotating the Alvarez lens pair. As a result, our solution can correct any prescription.

2. Method

2.1 Basic Principle of Alvarez Lenses

The Alvarez lens is a compact focus-tunable device enabled by its in-plane displacement mechanism. Generally, when two lenses with complementary profiles are displaced along an axis (assumed to be the x-axis), the resulting equivalent wavefront change is given by Eq. (1).

$$W(x,y) = T(x+\Delta x, y) - T(x-\Delta x, y) = 2\Delta x \cdot \frac{\partial T(x,y)}{\partial x}, \quad (1)$$

where $T(x, y)$ is the surface profile of the lens.

Thus, if a defocusing term in the form of a quadratic wavefront given in Eq. (2) is needed, the lens surface profile can be derived through integral along the x-axis, as Eq. (3) provides.

$$W(x,y) = x^2 + y^2 \quad (2)$$

$$T(x,y) = \frac{1}{3}x^3 + xy^2 \quad (3)$$

The above is fundamental to traditional one-dimensionally displaced Alvarez lenses, where the equivalent focal length—the only adjustable parameter, is determined by the displacement distance along the x-axis (see the Δx term in Eq. (1)).

2.2 Proposed Design

Our goal is to configure the ratio between defocus and astigmatism arbitrarily. Thus, first consider the wavefront expression of vertical astigmatism given by Eq. (4). By combining the defocus term in Eq. (2), the wavefront of an arbitrary hybrid of defocus and astigmatism is given by Eq. (5).

$$W(x,y) = y^2 \quad (4)$$

$$W(x,y) = k_1(x^2+y^2) + k_2 y^2 = K_1 x^2 + K_2 y^2, \quad (5)$$

where the magnitudes of defocus and astigmatism are configured through the coefficients K_1 and K_2.

The conventional 1D Alvarez lenses cannot arbitrarily configure K_1 and K_2. This study novelly considers 2D displacement by setting the displacement direction as an additional variable besides the displacement distance. When two complementary profiles are displaced for a distance Δl in a specific direction ($\cos\alpha$, $\cos\beta$), the equivalent wavefront change is given by Eq. (6). Naturally, the partial derivative in the conventional Alvarez principle is extended to the directional derivative along the displacement direction.

$$\begin{aligned} W(x,y) &= T(x+\Delta l\cdot\cos\alpha, y+\Delta l\cdot\cos\beta) - T(x,y) \\ &= \left(\frac{\partial T}{\partial x}\cos\alpha + \frac{\partial T}{\partial y}\cos\beta\right)\cdot\Delta l \end{aligned} \quad (6)$$

In Eq. (6), the direction cosines are critical because their ratio can be arbitrarily set through the displacement direction. Therefore, the proposed Alvarez lens profile is obtained by respectively integral on the two quadratic terms in the wanted wavefront, as the cubic profile described by Eq. (7). After displacing the two complementary surfaces, the equivalent wavefront is a summation

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of two quadratic terms with a ratio set by the direction cosines, as given by Eq. (8). The ratio between cosα and cosβ determines the ratio between defocus and astigmatism; the displacement distance Δl corresponds to the magnitude of the wavefront. Eq. (8) can also be interpreted as a combination of a spherical lens and a cylindrical lens by considering that two crossed cylindrical lenses are equivalent to a spherical lens.

$$T(x,y) = A(x^3 + y^3) + D, \tag{7}$$

where A is the coefficient that determines the ratio of displacement to optical power; D is the center thickness of the lens, which is employed to ensure the thinnest portion has sufficient mechanical strength.

$$W(x,y) = (\cos\alpha \cdot x^2 + \cos\beta \cdot y^2) \cdot A \cdot \Delta l \tag{8}$$

In summary, Figs. 1(a) and (b) illustrate the surface profile of the traditional Alvarez lens, namely Eq. (3), and its 1D displacement along the x-axis. Figs. 1(c) and (d) show the surface profile of the proposed 2D Alvarez lenses, namely Eq. (7), and its 2D displacement.

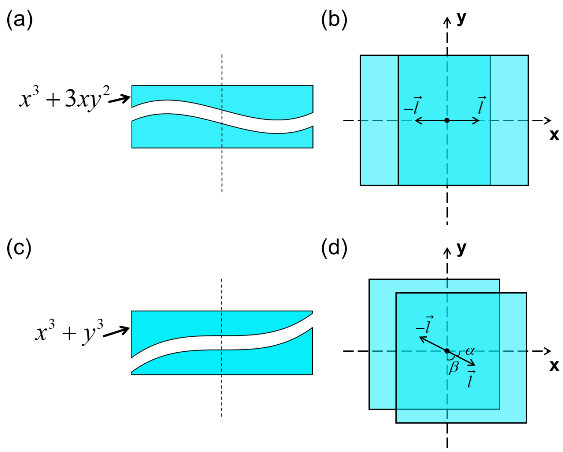

Fig. 1. (a) The traditional Alvarez lenses and (b) their top view showing a 1D displacement. (c) The proposed Alvarez lenses and (d) the top view showing a 2D displacement.

By integrating the proposed Alvarez lenses into a NED, defocused and astigmatic vision, or their combination, can be corrected. Furthermore, if the axial direction of astigmatism changes, we can rotate the Alvarez lens pair to adapt to the axial orientation without affecting the defocus correction.

3. Experimental Verification

Fig. 2(a) shows our experimental verification setup. An Apple Vision Pro is employed as the NED. A cylindrical lens (f = 250 mm, corresponding to 4-diopter astigmatism) is placed before a cellphone camera to imitate an astigmatic eye. Defocus (myopia or hyperopia) is generated by adjusting the camera's focusing state. The proposed 2D Alvarez lenses are fabricated and inserted between the NED and the camera, as shown in Fig. 2(b).

Four various refractive errors are investigated: (i) 3.3-diopter myopia, (ii) 4-diopter astigmatism with an axis at 0°, (iii) the myopia plus astigmatism, and (iv) the myopia plus the astigmatism but rotate the axis to 45°. Eq. (9) provides optical path lengths needing compensation for the first three cases. The last case equals the second one after the coordinate system is rotated for 45°. Based on Eq. (8), the displacement direction of the Alvarez lenses can be thus determined: (i) 45° to the x-axis, (ii) y-axis, (iii) 69.94° to the x-axis, and (iv) 69.94° to the new x-axis that has been rotated for 45°. The displacement distances are determined according to the magnitude of Φ_1 and Φ_2 in Eq. (9).

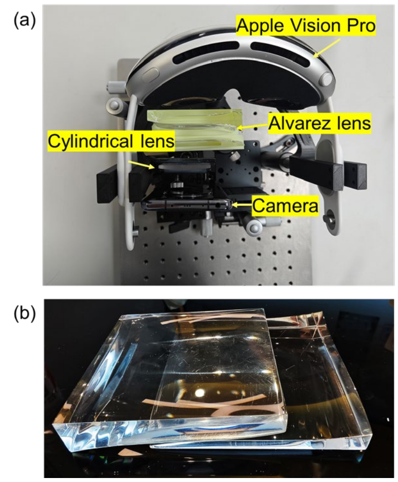

Fig. 2. (a) Experimental setup. (b) The fabricated Alvarez lens pair.

$$\begin{aligned} &(i)\ W_1 = -\Phi_1 \frac{x^2 + y^2}{2} \\ &(ii)\ W_2 = -\Phi_2 \frac{y^2}{2} \\ &(iii)\ W_3 = W_1 + W_2 \end{aligned}, \tag{9}$$

where Φ_1 and Φ_2 are optical powers of the myopia and astigmatism, equaling 3.3 and 4 diopters, respectively.

Fig. 3 shows experimental results observed through the Apple Vision Pro corresponding to the four refractive errors. Images corrected by the Alvarez lenses are compared with those before correction and the normal vision. The results demonstrate effectively retrieved image quality.

Furthermore, for quantitative assessment of image retrieval, a square pattern is presented to test modulation transfer functions (MTFs) using the slanted-edge method [9], as Fig. 4 shows. These images are captured using a Nikon Z8 with a 35mm F1.4 DG DN lens from SIGMA and sent to an MTF calculation toolkit.

In our assessment, the native MTF of the Apple Vision Pro exceeds 0.2 at 30 pixels per degree (PPD) (the blue curves in Fig. 4). It should be emphasized that this value may slightly underestimate the actual performance of the Apple Vision Pro due to the in-house experimental setup. Nevertheless, this measurement is still a good baseline for comparing MTFs with refractive errors and corrected ones. In the presence of the four refractive errors, MTF values show a significant degradation, even below 0.2 at 2 PPD (the green curves in Fig. 4). After correcting using the proposed Alvarez lenses, MTFs are retrieved to above 0.2 at 24 PPD (the red curves in Fig. 4), which are close to the native MTF of the Vision Pro (above 0.2 at 30 PPD). While the slight gap concerning normal vision is acceptable in practical use, the gap may be attributed to the chromatic aberration induced by the Alvarez lenses, which can be suppressed using low-dispersion materials.

4. Conclusion

This study proposed a novel Alvarez lens with 2D displacement. The direction and distance of the displacement provide an additional degree of freedom compared with traditional 1D Alvarez lenses. We utilized the 2D displacement to correct arbitrary hybrid defocus and astigmatism, experimentally verified on an Apple Vision Pro.

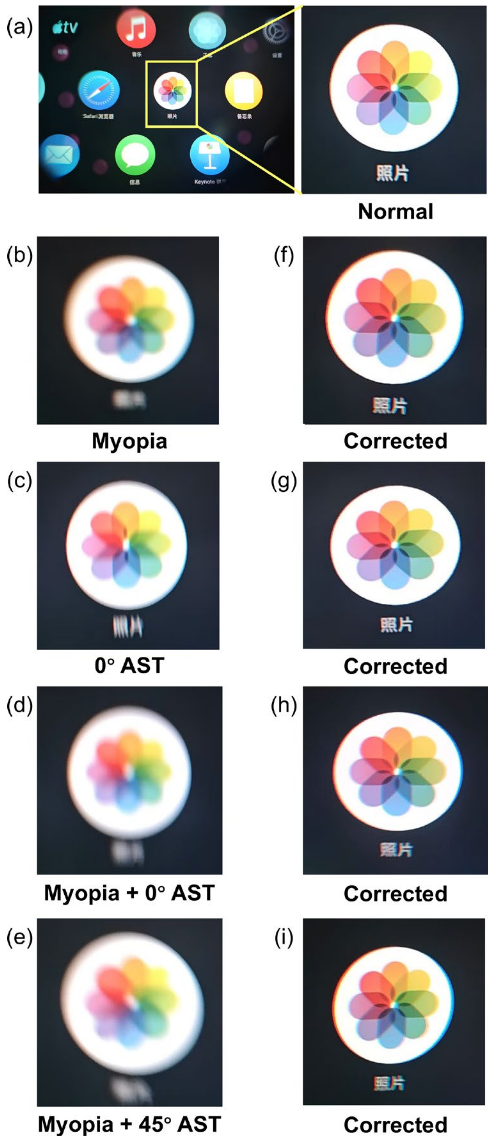

Fig. 3. Images through the Apple Vision Pro taken by the camera: (a) normal vision; (b) myopia; (c) 0° astigmatism (AST); (d) myopia plus 0° astigmatism; (e) myopia plus 45° astigmatism; (f)-(i): images corrected by the proposed Alvarez lenses.

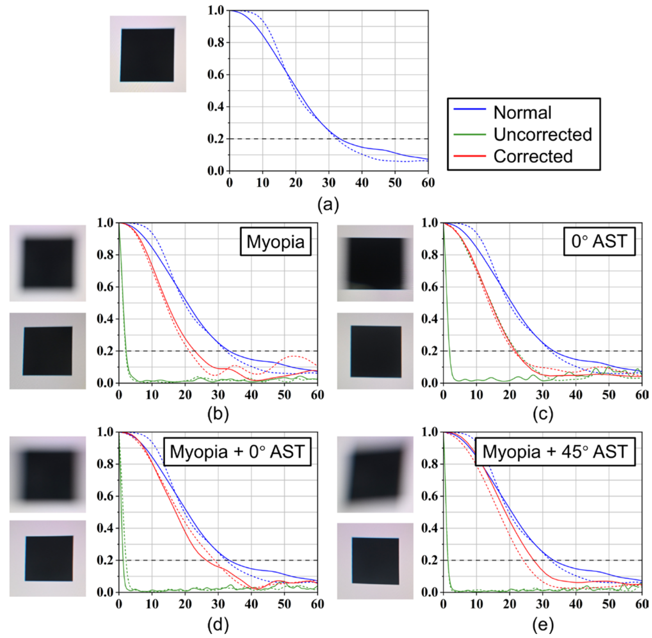

Fig. 4. A square pattern presented through the Apple Vision Pro and MTFs measured based on it. (a) Normal vision; (b) myopia; (c) 0° astigmatism (AST); (d) myopia plus 0° astigmatism; (e) myopia plus 45° astigmatism (abscissa: spatial frequency in PPD; ordinate: MTF value; solid lines: tangential; dashed lines: sagittal).

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